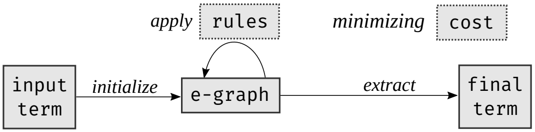
Sketch-Guided Equality Saturation

Thomas Kœhler   Phil Trinder   Michel Steuwer

University of Glasgow   The University of Edinburgh

EGRAPHS workshop, PLDI, San Diego — June 2022

Equality Saturation



- Optimize programs by efficiently exploring many possible rewrites
- Many successful applications sparked from the recent `egg` library

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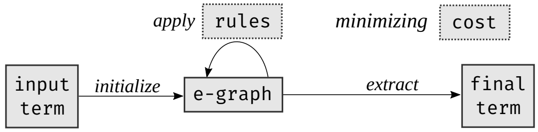

- Optimize programs by efficiently exploring many possible rewrites
- Many successful applications sparked from the recent `egg` library

Some optimizations remain out of reach as the e-graph grows too big

Case Study

Matrix Multiplication Optimizations for CPU:

- transform loops

  blocking, permutation, unrolling
- change data layout
- add parallelism

  vectorization, multi-threading

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- add parallelism

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Space of equivalent programs to consider is huge

Case Study

- Rewritten language: RISE, a functional array language

Matrix Multiplication in RISE:

```
def mm a b =
  map (λaRow.                    | for aRow in a:
    map (λbCol.                  |   for bCol in transpose(b):
      dot aRow bCol)             |     ... = dot(aRow, bCol)
      (transpose b)) a

def dot xs ys =
  reduce + 0                     | for (x, y) in zip(xs, ys):
    (map (λ(x, y). x × y)        |   acc += x × y
      (zip xs ys))
```

Case Study

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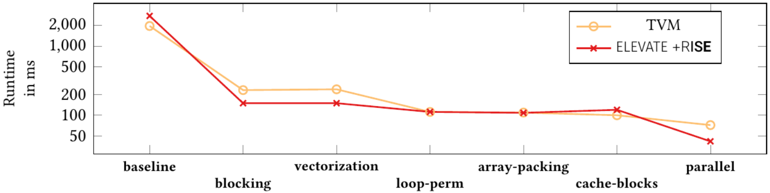
```

RISE is designed for optimization via term rewriting

Case Study

- Prior work optimizes MM in RISE using rewriting strategies
  - manually control when to apply each rewrite rule
  - talk by Michel Steuwer on Friday, 15h30 at Cockatoo



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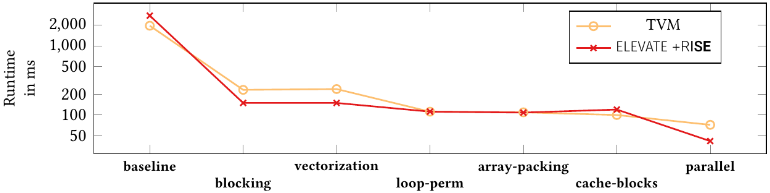

Great performance, but requires manual rewrite ordering

Case Study

- Achieve the same 7 optimization goals with equality saturation?[1]

goal	found?	runtime	RAM
baseline	✓	0.5s	0.02 GB
blocking	✓	>1h	35 GB
vectorization	✗	>1h	>60 GB
loop-perm	✗	>1h	>60 GB
array-packing	✗	35mn	>60 GB
cache-blocks	✗	35mn	>60 GB
parallel	✗	35mn	>60 GB

- Most goals are not found before exhausting 60 GB.
- For comparison, rewriting strategies take <2s and <1GB.

[1] on Intel Xeon E5-2640 v2

Case Study

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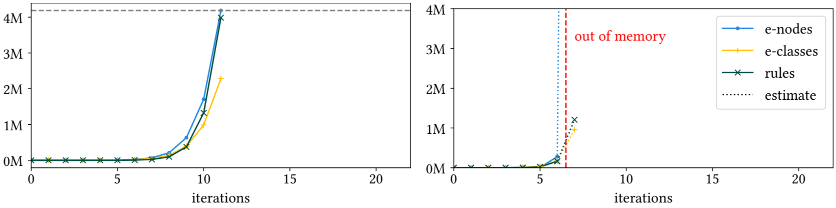
Standard equality saturation does not scale to this optimization space

[1] on Intel Xeon E5-2640 v2

E-Graph Evolution



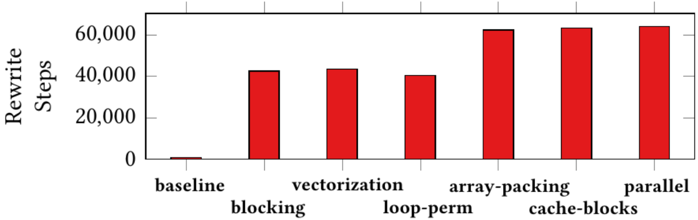
(a) *blocking*, found: ✓ (b) *parallel*, found: ✗

Two difficulties:

1. Long rewrite sequences $\implies$ many iterations are required
2. Explosive combination of rewrite rules $\implies$ exponential growth
   - millions of e-nodes and e-classes in less than 10 iterations
   - worse for *parallel*, memory is exhausted in the 7th iteration

Difficulty 1. Long Rewrite Sequences

Prior work *(not shortest path)*:



Difficulty 2. Explosive Combinations of Rewrite Rules

Two example rules that quickly generate many possibilities:

split-join:

```
    map f x                  | for m:
                             |  ... = f(...)
      ↦
join
  (map                       | for m / n:
    (map f)                  |  for n:
    (split n x))             |   ... = f(...)
```

transpose-around-map-map:

```
    map                      | for m:
      (map f) x              |  for n:
                             |   ... = f(...)
        ↦
transpose
  (map                       | for n:
    (map f)                  |  for m:
    (transpose x))           |   ... = f(...)
```

To overcome these difficulties, we came up with *sketch-guided equality saturation*

Sketch-Guided Equality Saturation

Observation:

- The *shape* of the optimised program is often used to explain optimizations:

```
for m:
 for n:
  for k:
   ..
```

$\mapsto^*$

```
for m / 32:
 for n / 32:
  for k / 4:
   for 4:
    for 32:
     for 32:
      ..
```

Sketch-Guided Equality Saturation

Observation:

- The *shape* of the optimised program is often used to explain optimizations:

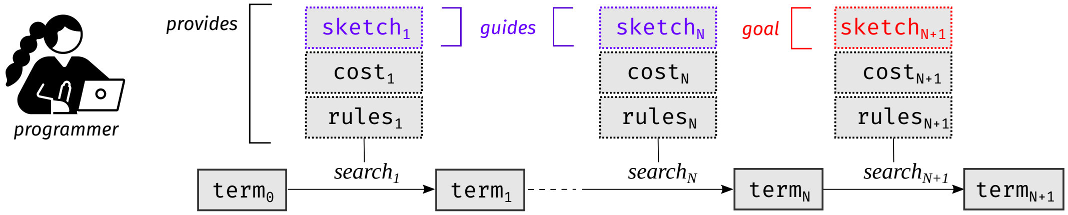
```
for m:
 for n:
  for k:
   ..
```

$\mapsto^*$

```
for m / 32:
 for n / 32:
  for k / 4:
   for 4:
    for 32:
     for 32:
      ..
```

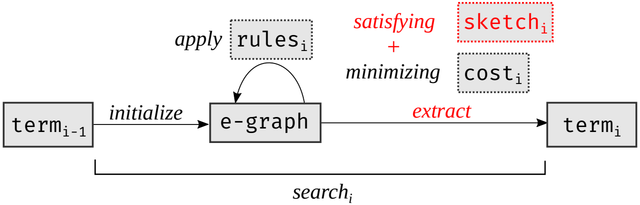
Explanatory shapes can be formalized as sketches and used to guide rewriting

Sketch-Guided Equality Saturation



- Factors an unfeasible search into a sequence of feasible ones:
  1. Break long rewrite sequences
  2. Isolate explosive combinations of rewrite rules

Sketch-Satisfying Equality Saturation



- Terminates as soon as a program satisfying the sketch is found

Sketches

- *Sketches* are program patterns that leave details unspecified

baseline sketch:

```
containsMap(m,                      | for m:
 containsMap(n,                     |  for n:
  containsReduceSeq(k,              |   for k:
   containsAddMul)))                |     .. + .. × ..
```

- Abstractions defined in terms of smaller building blocks:

```
def containsAddMul: Sketch =
  contains(app(app(+, ?), contains(×)))
```

Sketches

- *Sketches* are program patterns that leave details unspecified

baseline sketch:

```
containsMap(m,                  | for m:
 containsMap(n,                 |  for n:
  containsReduceSeq(k,          |   for k:
   containsAddMul)))            |     .. + .. × ..
```

- A sketch `s` is satisfied by a set of terms `R(s)`:

```
def containsAddMul: Sketch =
  contains(app(app(+, ?), contains(×)))

R(containsAddMul) = { R(app(app(+, ?), contains(×))) } ∪
  { F(t1, .., tn) | ∃ti ∈ R(containsAddMul) }
R(app(app(+, ?), contains(×))) = { app(app(+, t1), t2) | t2 ∈ R(contains(×)) }
R(contains(×)) = { × } ∪ { F(t1, .., tn) | ∃ti ∈ R(contains(×)) }
```

Sketches

- *Sketches* are program patterns that leave details unspecified

baseline sketch:

```
containsMap(m,                  | for m:
 containsMap(n,                 |  for n:
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   containsAddMul)))            |     .. + .. × ..
```

blocking sketch:

```
containsMap(m / 32,             | for m / 32:
 containsMap(n / 32,            |  for n / 32:
  containsReduceSeq(k / 4,      |   for k / 4:
   containsReduceSeq(4,         |    for 4:
    containsMap(32,             |     for 32:
     containsMap(32,            |      for 32:
      containsAddMul))))))      |       .. + .. × ..
```

Sketches

- *Sketches* are program patterns that leave details unspecified

baseline sketch:

```
containsMap(m,                      | for m:
 containsMap(n,                     |  for n:
  containsReduceSeq(k,              |   for k:
   containsAddMul)))                |     .. + .. × ..
```

sketch guide:

how to split the loops before reordering them?

```
containsMap(m / 32,                 | for m / 32:
 containsMap(32,                    |  for 32:
  containsMap(n / 32,               |   for n / 32:
   containsMap(32,                  |    for 32:
    containsReduceSeq(k / 4,        |     for k / 4:
     containsReduceSeq(4,           |      for 4:
      containsAddMul))))))          |       .. + .. × ..
```

blocking sketch:

```
containsMap(m / 32,                 | for m / 32:
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  containsReduceSeq(k / 4,          |   for k / 4:
   containsReduceSeq(4,             |    for 4:
    containsMap(32,                 |     for 32:
     containsMap(32,                |      for 32:
      containsAddMul))))))          |       .. + .. × ..
```

Evaluation

- Equality Saturation without Sketch Guides[2]:

goal	found?	runtime	RAM
baseline	✓	0.5s	0.02 GB
blocking	✓	>1h	35 GB
+ 5 others	✗	>35mn	>60 GB

- Sketch-Guided Equality Saturation[3]:

goal	sketch guides	found?	runtime	RAM
baseline	0	✓	0.5s	0.02 GB
blocking	1	✓	7s	0.3 GB
+ 5 others	2-3	✓	≤7s	≤0.5 GB

[2] Intel Xeon E5-2640 v2

[3] AMD Ryzen 5 PRO 2500U

Evaluation

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Sketch-guided equality saturation finds all 7 optimization goals

[2] Intel Xeon E5-2640 v2

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Evaluation

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582x

116x

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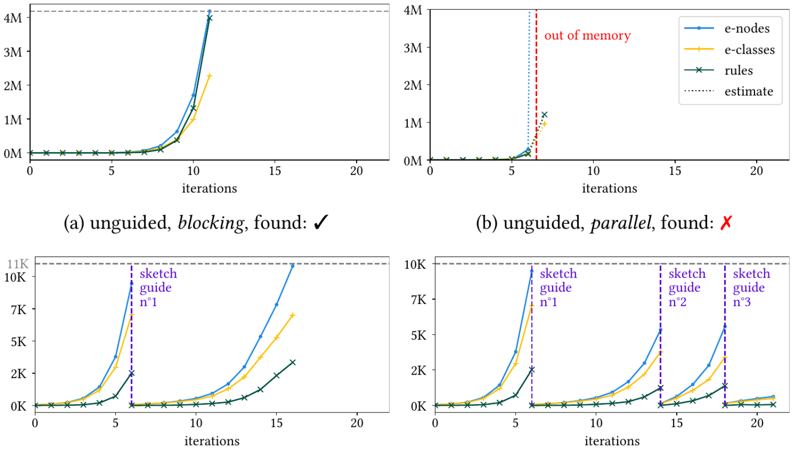

(a) unguided, *blocking*, found: ✓

(b) unguided, *parallel*, found: ✗

(c) sketch-guided, *blocking*, found: ✓

(d) sketch-guided, *parallel*, found: ✓

Evaluation

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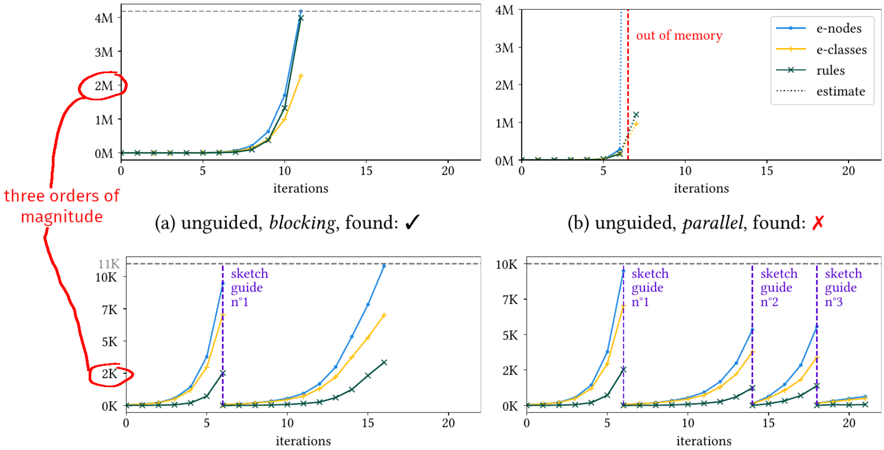

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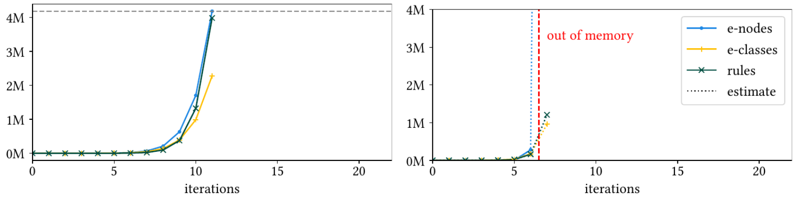

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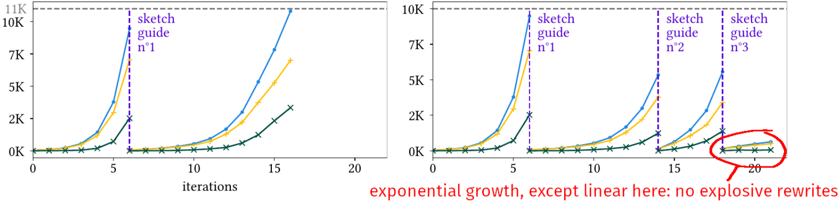

(c) sketch-guided, *blocking*, found: ✓ (d) sketch-guided, *parallel*, found: ✓

Evaluation

Sketches vs Full Program

goal	sketch guides	sketch goal	sketch sizes	program size
blocking	*split*	$reorder_1$	7	90
vectorization	*split* + $reorder_1$	$lower_1$	7	124
loop-perm	*split* + $reorder_2$	$lower_2$	7	104
array-packing	*split* + $reorder_2$ + *store*	$lower_3$	7-12	121
cache-blocks	*split* + $reorder_2$ + *store*	$lower_4$	7-12	121
parallel	*split* + $reorder_2$ + *store*	$lower_5$	7-12	121

- each sketch corresponds to a logical transformation step
- sketches elide around 90% of the program
- intricate details such as array reshaping patterns are not specified (e.g. `split`, `join`, `transpose`)

Future Work

- **Combine with precise control of rewriting strategies?**
  - equality saturation as a rewriting strategy
    - other talk at 11h30: *Equality Saturation as a Tactic for Proof Assistants*
  - using rewriting strategies or tactics inside equality saturation?
- **More diverse applications** and languages, maybe theorem proving?
- Focused growth? (rewrite rule scheduling, heuristics, pruning, etc)
- How to select effective sketch guides, sets of rules and cost models in general?
- More powerful sketch language, reusing sketches across diverse programs?
- Can we synthesize sketch guides from a sketch goal?
- Interactive optimization assistant?

Conclusion

We propose:

- *sketches* to guide rewriting
- *sketch-guided equality saturation*, a novel, semi-automatic optimization technique

https://arxiv.org/abs/2111.13040

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thomas.koehler@thok.eu
thok.eu

Thanks!
We are open to collaboration!

rise-lang.org
elevate-lang.org

Sketch Definition

$$S \ ::= \ ? \ \mid \ F(S, .., S) \ \mid \ contains(S)$$

$$\begin{aligned}
R(?) &= T = \{F(t_1, .., t_n)\} \\
R(F(s_1, .., s_n)) &= \{F(t_1, .., t_n) \mid t_i \in R(s_i)\} \\
R(contains(s)) &= R(s) \cup \{F(t_1, .., t_n) \mid \exists t_i \in R(contains(s))\}
\end{aligned}$$

```
def containsMap(n: NatSketch, f: Sketch): Sketch =
  contains(app(map :: ?t → n.?dt → ?y, f))

def containsReduceSeq(n: NatSketch, f: Sketch): Sketch =
  contains(app(reduceSeq :: ?t → ?t → n.?dt → ?t, f))

def containsAddMul: Sketch =
  contains(app(app(+, ?), contains(×)))
```

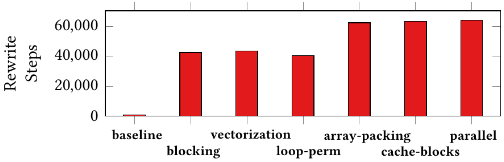
MM Blocking

```
map (λaRow.                  | for m:
  map (λbCol.                |  for n:
    dot aRow bCol)           |   for k:
    (transpose b)) a         |    ...
```

$\mapsto^*$

```
join (map (map join) (map transpose
  map                                  | for m / 32:
   (map λx2.                           |  for n / 32:
     reduceSeq (λx3. λx4.              |   for k / 4:
       reduceSeq λx5. λx6.             |    for 4:
         map                           |     for 32:
          (map (λx7.                   |      for 32:
            (fst x7) + (fst (snd x7)) ×
               (snd (snd x7)))
             (map (λx7. zip (fst x7) (snd x7))
               (zip x5 x6)))
        (transpose (map transpose
        (snd (unzip (map unzip map (λx5.
          zip (fst x5) (snd x5))
          (zip x3 x4)))))))
       (generate (λx3. generate (λx4. 0)))
       transpose (map transpose x2))
   (map (map (map (map (split 4))))
     (map transpose
       (map (map (λx2. map (map (zip x2)
         (split 32 (transpose b)))))
          split 32 a))))))
```

Prior work *(not shortest path)*:



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Deciding How to Apply Rewrite Rules

Fully automated search?

e.g. heuristic search,
equality saturation, …

This is fully **automated**!

The search takes forever ..

Why is the performance so bad in this case?

I wish I could have control over the optimizations!

Manually written recipe?

e.g. Halide/TVM schedules,
Elevate strategies, …

I have full **control**!

This was more difficult to write than I expected!

How can I generalize this recipe to other cases?

Guided search!

I can combine **control and automation**!

Rewriting Strategies

- programmers describe optimizations as compositions of rewrite rules
- *blocking*:

```
def blocking = ( baseline ‘;‘
  tile(32,32)       ‘@‘ outermost(mapNest(2))   ‘;;‘
  fissionReduceMap ‘@‘ outermost(appliedReduce)‘;;‘
  split(4)          ‘@‘ innermost(appliedReduce)‘;;‘
  reorder(List(1,2,5,6,3,4)))
```

\+ empowers programmers to manually control the rewrite process

\+ `tile`, `split`, `reorder` are not built-in but programmer-defined

Bastian Hagedorn, Johannes Lenfers, Thomas Koehler, Xueying Qin, Sergei Gorlatch, and Michel Steuwer. “Achieving high-performance the functional way: a functional pearl on expressing high-performance optimizations as rewrite strategies”. In: ICFP (2020)

Rewriting Strategies

- programmers describe optimizations as compositions of rewrite rules
- *blocking*:

```
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  reorder(List(1,2,5,6,3,4)))
```

- requires programmers to order all rewrite steps deterministically
- strategies are often program-specific and complex to implement
- transformed program is hidden state that needs to be reasoned about

Handwritten Matrix Multiplication

```
for (int im = 0; im < m; im++) {
  for (int in = 0; in < n; in++) {
    float acc = 0.0f;
    for (int ik = 0; ik < k; ik++) {
      acc += a[ik + (k * im)] * b[in + (n * ik)];
    }

    output[in + (n * im)] = acc;
  }
}
```

Optimised program on the right:

- \+ 110× faster runtime

  Intel i5-4670K CPU

- \- 6× more lines of code where things can go wrong

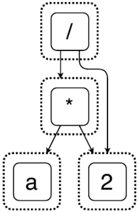
  threads, SIMD, index computations

- \- hardware specific (not portable)

```
float aT[n * k];
#pragma omp parallel for
for (int in = 0; in < (n / 32); in = 1 + in) {
  for (int ik = 0; ik < k; ik = 1 + ik) {
    #pragma omp simd
    for (int jn = 0; jn < 32; jn = 1 + jn) {
      aT[(ik + ((32 * in) * k)) + (jn * k)] = a[(jn + (32 * in)) + (ik * n)];
    }
  }
}
#pragma omp parallel for
for (int im = 0; im < (m / 32); im = 1 + im) {
  for (int in = 0; in < (n / 32); in = 1 + in) {
    float tmp1[1024];
    for (int jm = 0; jm < 32; jm = 1 + jm) {
      for (int jn = 0; jn < 32; jn = 1 + jn) {
        tmp1[jn + (32 * jm)] = 0.0f;
      }
    }
    for (int ik = 0; ik < (k / 4); ik = 1 + ik) {
      for (int jm = 0; jm < 32; jm = 1 + jm) {
        float tmp2[32];
        for (int jn = 0; jn < 32; jn = 1 + jn) {
          tmp2[jn] = tmp1[jn + (32 * jm)];
        }
        #pragma omp simd
        for (int jn = 0; jn < 32; jn = 1 + jn) {
          tmp2[jn] += (a[((4 * ik) + ((32 * im) * k)) + (jm * k)] * aT[((4 * ik) + ((32 * in) * k)) + (jn * k)]);
        }
        #pragma omp simd
        for (int jn = 0; jn < 32; jn = 1 + jn) {
          tmp2[jn] += (a[((1 + (4 * ik)) + ((32 * im) * k)) + (jm * k)] *
            aT[((1 + (4 * ik)) + ((32 * in) * k)) + (jn * k)]);
        }
        #pragma omp simd
        for (int jn = 0; jn < 32; jn = 1 + jn) {
          tmp2[jn] += (a[((2 + (4 * ik)) + ((32 * im) * k)) + (jm * k)] *
            aT[((2 + (4 * ik)) + ((32 * in) * k)) + (jn * k)]);
        }
        #pragma omp simd
        for (int jn = 0; jn < 32; jn = 1 + jn) {
          tmp2[jn] += (a[((3 + (4 * ik)) + ((32 * im) * k)) + (jm * k)] *
            aT[((3 + (4 * ik)) + ((32 * in) * k)) + (jn * k)]);
        }
        for (int jn = 0; jn < 32; jn = 1 + jn) {
          tmp1[jn + (32 * jm)] = tmp2[jn];
        }
      }
    }
    for (int jm = 0; jm < 32; jm = 1 + jm) {
      for (int jn = 0; jn < 32; jn = 1 + jn) {
        output[((jn + ((32 * im) * n)) + (32 * in)) + (jm * n)] = tmp1[jn + (32 * jm)];
      }
    }
  }
}
```

E-Graph Example

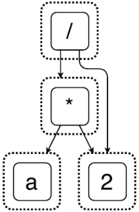
$$(a * 2)/2 \longrightarrow^{*} a$$



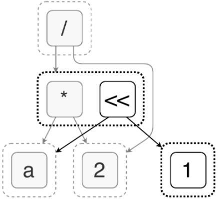
$(a * 2)/2$

E-Graph Example

$$(a * 2)/2 \longrightarrow^* a$$



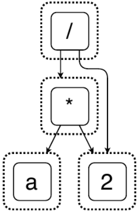
$(a * 2)/2$

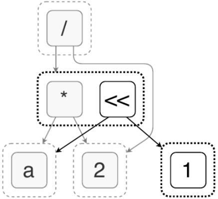


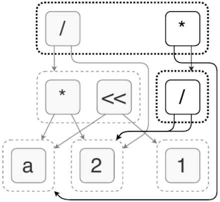
$x * 2 \longrightarrow x \ll 1$

E-Graph Example

$$(a * 2)/2 \longrightarrow^* a$$







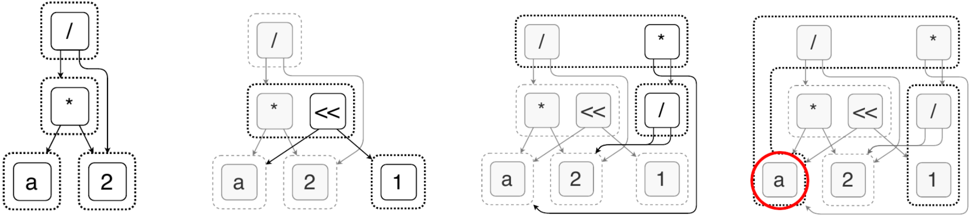
$(a * 2)/2$

$x * 2 \longrightarrow x \ll 1$

$(x * y)/z \longrightarrow x * (y/z)$

E-Graph Example

$$(a * 2)/2 \longrightarrow^* \color{red}{a}$$



$(a * 2)/2$

$x * 2 \longrightarrow x \ll 1$

$(x * y)/z \longrightarrow x * (y/z)$

$x/x \longrightarrow 1$

$x * 1 \longrightarrow x$

cost = term size